

IFUP-TH 2/94
HUTP-93/A038

The $b \rightarrow s\gamma$ decay revisited

Giancarlo Cella^a, Giuseppe Curci^b

*Dipartimento di Fisica dell'Università, Piazza Torricelli 2, I-56126 Pisa, Italy, and
Istituto Nazionale di Fisica Nucleare, Via Livornese 582/a I-56010 S. Piero a Grado
(Pisa), Italy*

Giulia Ricciardi^c

Lyman Laboratory of Physics, Harvard University, Cambridge, MA 02138

Andrea Vicerè^d

*Istituto Nazionale di Fisica Nucleare, Piazza Torricelli 2, I-56100 Pisa, Italy
(February 1, 2008)*

Abstract

In this work we compute the leading logarithmic corrections to the $b \rightarrow s\gamma$ decay in a dimensional scheme which does not require any definition of the γ_5 matrix. The scheme does not exhibit inconsistencies and it is therefore a viable alternative to the t'Hooft Veltman scheme, particularly in view of the next-to-leading computation. We confirm the recent results of Ciuchini et al.

PACS numbers: 11.10.Gh, 12.38.Bx, 13.20.Jf

Typeset using REVTeX

I. INTRODUCTION

The $b \rightarrow s\gamma$ decay has received considerable interest in the last years, thanks to its sensitivity to physics beyond the Standard Model.

On the experimental side, the limit for the inclusive decay based on the first observation of the $B \rightarrow K^*\gamma$ transition by the CLEO collaboration [1,2], are compatible with the SM predictions, taking into account leading QCD corrections.

On the theoretical side, after the first computations of the decay amplitudes [3–7] and the confirmation of the results of [3] given by us in [8], several improvements have occurred.

In [9,10] the analysis has been extended to include all the relevant operators that mix under QCD corrections and contribute to the leading logarithmic corrections.

In [11,12] the long-standing problem of the differences between the results obtained in [3,8] and in [4,5] has been solved, by showing that the anomalous dimension matrix is scheme dependent even at the leading order. This effect is compensated by the matrix elements of 4 fermion operators, and the physical results are scheme independent. A similar analysis limited to a subset of the relevant operators is also presented in [13].

In [14] it has been performed an analysis of all the sources of uncertainty in the leading logarithmic order computation (LLO) and it has been shown that the inclusion of the next to leading order corrections (NLO) can reduce the uncertainty from 25% to less than 10%.

In view of these considerations we think that it is time to undertake a NLO computation and in this paper we reconsider the LLO computation with a technique that we think particularly apt to this purpose. As a byproduct, we confirm the recent results of Ciuchini et al. [11,12].

Dimensional methods differ in the treatment of the γ_5 matrix: to our knowledge, the only method that does not exhibit any inconsistency is the t’Hooft Veltman scheme (HV) [15]. Unfortunately, this method is very cumbersome and in literature simplified versions have been used, like naive dimensional regularization (NDR) or dimensional reduction (DRED) [16]. It has been shown ([17,18,11,12] and references therein) that NDR, DRED, HV methods give the same physical results in a large class of radiative-correction computations (up to two loops) where no trace ambiguities arise; on the other hand it is well known that inherent pathologies of NDR and DRED schemes are present at three-loops level [19]. We are therefore interested in a *practical* approach to the evaluation of radiative corrections, as proposed in [20], avoiding the extension of the γ_5 matrix to $d \neq 4$. This method appears to be free of ambiguities, and in our opinion presents less complications than the HV scheme, especially when performing computations by symbolic manipulation programs.

The plan of the letter is as follows: in section (II) we describe the scheme we have used, recalling the results in [20], then in section (III) we show how we have

applied the method to the $b \rightarrow s\gamma$ process.

II. STRATEGY

We aim to evaluate the amplitude for a decay process induced by the exchange of virtual particles, such as the flavor changing neutral current processes.

The integration of the heavy degrees of freedom (in our case, W boson and t quark) leaves an effective hamiltonian, whose matrix elements in the background of the “light” theory determine the decay amplitude at the lowest order in the heavy-mass expansion

$$\langle f | i \rangle_{\text{full}} = \frac{1}{M^2} \sum_j C_j(M, \mu) \langle f | O_j | i \rangle_{\text{light}}(\mu) + O\left(\frac{1}{M^4}\right). \quad (1)$$

The coefficients of the operators in the effective hamiltonian are determined matching the two sides of (1) in perturbation theory at a scale of the order of the heavy masses. At $\mu \simeq M$ all the QCD large logarithms originate in the operator matrix elements of the effective theory and the improved perturbation theory allows to resum them systematically. The RG invariance of the l.h.s. of (1) is exploited, together with the calculable μ dependence of the matrix elements, to evolve down the r.h.s. at values of μ comparable with the scales of the external states; this amounts to “transfer” the large logarithms from the matrix elements to the coefficients.

The key element is therefore the evaluation of the anomalous dimension matrix of the operators, which determines their μ evolution, together with the “matching” conditions at the M scale and the matrix elements at the lower end of the evolution. This procedure is in general dependent on the regularization and renormalization scheme chosen for the operators.

Now what is important for our discussion is the freedom to define the d dimensional extension of the operators of the effective hamiltonian: as in [20] we will use this freedom to simplify the computation.

Let us first observe that the general structure of the effective hamiltonian for the $b \rightarrow s\gamma$ decay contains chiral projectors $P_{L/R} = 1/2(1 \mp \gamma_5)$; therefore a definition of the γ_5 matrix is required in d dimensions. We have

$$\mathcal{H}_{\text{eff}}^1 = \sum_i C_i^R R_i + \sum_i C_i^{LL} (L \otimes L)_i + \sum_i C_i^{LR} (L \otimes R)_i, \quad (2)$$

where R_i stand for the magnetic momentum operators, while $(L \otimes L)_i$, $(L \otimes R)_i$ are the current-current operators: the field content is immaterial at this level. Consider on the other hand the effective hamiltonian obtained with an exchange $\gamma_5 \rightarrow -\gamma_5$

$$\mathcal{H}_{\text{eff}}^2 = \sum_i C_i^R L'_i + \sum_i C_i^{LL} (R \otimes R)'_i + \sum_i C_i^{LR} (R \otimes L)'_i \quad (3)$$

where we intend that the operator L'_i is equal to R_i apart the chiral projector, and analogously for the other operators.

Since QCD does not know the sign of γ_5 , the two hamiltonians will receive the same perturbative corrections, that is

$$\begin{aligned} \mathcal{D}N[L \otimes L_i] + \gamma_{ij}^{LLR} N[R_j] + \gamma_{ij}^{LLLL} N[(L \otimes L)_j] + \gamma_{ij}^{LLLR} N[(L \otimes R)_j] &= 0 \\ \mathcal{D}N[(R \otimes R)_i] + \gamma_{ij}^{LLR} N[L'_j] + \gamma_{ij}^{LLLL} N[(R \otimes R)_j] + \gamma_{ij}^{LLLR} N[(R \otimes L)_j] &= 0 \\ \dots, \end{aligned} \quad (4)$$

assuming the same renormalization scheme is used. The $N[O]$ symbol stands for the operator O renormalized by minimal subtraction.

Let us now take the linear combinations which are symmetric and anti-symmetric under the γ_5 sign flip

$$\mathcal{H}_{s/a} = \frac{1}{2} \left(\mathcal{H}_{\text{eff}}^1 \pm \mathcal{H}_{\text{eff}}^2 \right) . \quad (5)$$

The original hamiltonian, which determines the physical amplitude, is a combination of the symmetric and anti-symmetric hamiltonian; the key point is that in order to obtain the scaling properties of $\mathcal{H}_{\text{eff}}^1$ it is sufficient to know the QCD evolution of \mathcal{H}_s . In other words it is possible to choose the same renormalization scheme for \mathcal{H}_s and \mathcal{H}_a , and therefore determine, for instance, the evolution of an operator having chiral structure $(V - A) \otimes (V \mp A)$ from the evolution of $(V \otimes V) \pm (A \otimes A)$. In the case at hand we can choose a scheme for the symmetric part which is as simple as possible, and forget the anti-symmetric part.

Let us focus on the current-current operators; the extension of the symmetrized hamiltonian \mathcal{H}_s to d dimensions can be conveniently done reexpressing tensor products of γ matrices as elements of the Clifford d dimensional algebra, for instance

$$\frac{1}{2} [(V - A) \otimes (V - A) + (V + A) \otimes (V + A)] \equiv \gamma_\mu \otimes \gamma_\mu + \frac{1}{3!} \gamma_{\mu\nu\rho} \otimes \gamma_{\mu\nu\rho} . \quad (6)$$

More generally a complete basis in d dimensions is provided by the operators

$$\begin{aligned} \left(\gamma^{(n)} \otimes \gamma^{(n)} \right) &= \sum_{\mu, \mu_2, \dots, \mu_n} \gamma_{\mu_1, \mu_2, \dots, \mu_n} \otimes \gamma_{\mu_1, \mu_2, \dots, \mu_n} \\ \gamma_{\mu_1, \mu_2, \dots, \mu_n} &= \frac{1}{n!} \sum_{p \in \Pi_n} (-1)^p \gamma_{\mu_1} \gamma_{\mu_2} \dots \gamma_{\mu_n} . \end{aligned} \quad (7)$$

This extension to d -dimensions does not require any definition of a d dimensional γ_5 and therefore it avoids the complications due to the splitting between ε -dimensional space and 4-dimensional space. It also allows to treat relevant operators having $n \leq 4$ and evanescent operators having $n \geq 5$ at the same way.

Many technical details will be explained in a more extended work [21] where a detailed account of the computation and the full set of formulas needed in addition to the ones in [20] to implement the method will be given as a reference for future works.

III. THE LEADING LOG CORRECTIONS TO THE $B \rightarrow S\gamma$ HAMILTONIAN

Let us write down the effective on-shell hamiltonian for the $b \rightarrow s\gamma$, $b \rightarrow sg$, already used in [9,11], in euclidean notation:

$$\begin{aligned}
\mathcal{H}_1 &= \frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_i C_i Q_i \\
Q_1 &= (\bar{s}_\alpha c_\beta)_{V-A} (\bar{c}_\beta b_\alpha)_{V-A} \\
Q_2 &= (\bar{s}_\alpha c_\alpha)_{V-A} (\bar{c}_\beta b_\beta)_{V-A} \\
Q_{3,5} &= (\bar{s}_\alpha b_\alpha)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}_\beta q_\beta)_{V\pm A} \\
Q_{4,6} &= (\bar{s}_\alpha b_\beta)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}_\beta q_\alpha)_{V\pm A} \\
Q_7 &= \frac{(-ieQ_d)}{(4\pi)^2} m_b \bar{s} \sigma_{\mu\nu} (V+A) F_{\mu\nu} b \\
Q_8 &= \frac{(-ig_s)}{(4\pi)^2} m_b \bar{s} \sigma_{\mu\nu} (V+A) \hat{G}_{\mu\nu} b .
\end{aligned} \tag{8}$$

Following the steps outlined in the preceding section, we consider the on-shell basis, in d dimensions

$$\begin{aligned}
O_{(1,n)} &= \frac{1}{n!} (\bar{s}_\alpha \gamma^{(n)} c_\beta) (\bar{c}_\beta \gamma^{(n)} b_\alpha) \\
O_{(2,n)} &= \frac{1}{n!} (\bar{s}_\alpha \gamma^{(n)} c_\alpha) (\bar{c}_\beta \gamma^{(n)} b_\beta) \\
O_{(3,n)} &= \frac{1}{n!} (\bar{s}_\alpha \gamma^{(n)} b_\alpha) \sum_{q=u,d,s,c,b} (\bar{q}_\beta \gamma^{(n)} q_\beta) \\
O_{(4,n)} &= \frac{1}{n!} (\bar{s}_\alpha \gamma^{(n)} b_\beta) \sum_q (\bar{q}_\beta \gamma^{(n)} q_\alpha) \\
O_5 &= \frac{(-ieQ_d)}{(4\pi)^2} m_b \bar{s} \sigma_{\mu\nu} F_{\mu\nu} b \\
O_6 &= \frac{(-ig_s)}{(4\pi)^2} m_b \bar{s} \sigma_{\mu\nu} \hat{G}_{\mu\nu} b .
\end{aligned} \tag{9}$$

The symmetrized operators corresponding to the ones listed in (8) are defined as

$$\begin{aligned}
Q_1^s &= O_{(1,1)} + O_{(1,3)} \\
Q_2^s &= O_{(2,1)} + O_{(2,3)} \\
Q_{3/5}^s &= O_{(3,1)} \pm O_{(3,3)} \\
Q_{4/6}^s &= O_{(4,1)} \pm O_{(4,3)} \\
Q_7^s &= O_5 \\
Q_8^s &= O_6 .
\end{aligned} \tag{10}$$

The first step is to determine the anomalous dimension matrix of the operators listed in (9), by using algebraic identities for arbitrary n [22,23,20].

The next step is to determine the Renormalization Group evolution in d dimensional space:

$$\begin{aligned} \mathcal{D}N [O_{(i,n)}] + \sum_{j,m} \gamma_{(i,n),(j,m)} N [O_{(j,m)}] + \sum_k \gamma_{(i,n),k} N [O_k] &= 0 \\ \mathcal{D}N [O_n] + \sum_m \gamma_{nm} N [O_k] &= 0 \end{aligned} \quad (11)$$

where

$$\begin{aligned} \mathcal{D} &= \mu \frac{\partial}{\partial \mu} + \beta_{QCD} \frac{\partial}{\partial g_S} + \gamma_{m_b} m_b \frac{\partial}{\partial m_b} \\ \beta_{QCD} &= 2\varepsilon - g_s \frac{\alpha_s}{(4\pi)} \frac{11 C_A - 2 n_f}{3} \\ \gamma_{m_b} &= -\frac{\alpha_s}{(2\pi)} 3 C_F ; \end{aligned} \quad (12)$$

here and in the following C_A, C_F are the Casimir invariants of $SU(N_c)$, $C_A = N_c$, $C_F = (N_c^2 - 1) / (2 N_c)$, and n_f is the number of active flavors.

Note that the basis in (9) is infinite, but the sums in (11) run, at a definite order in the loop expansion, over a limited set of operators. For instance at one loop the sub-matrix γ of operators $O_{1/2,n}$ results as follows, omitting a factor $\alpha_s / (4\pi)$

$$\begin{aligned} &\begin{matrix} (1, n-2) & (1, n) & (1, n+2) \\ (1, n) \left(\begin{matrix} \frac{(6-n)(n-5)}{C_A} & 4 C_F ((1-n)(n-3)) & \frac{-((1+n)(2+n))}{C_A} \\ (2, n) \left(\begin{matrix} \frac{(n-6)(n-5)}{2} & 3(-2+4n-n^2) & \frac{(1+n)(2+n)}{2} \end{matrix} \right) \end{matrix} \right. \\ &\quad \begin{matrix} (2, n-2) & (2, n) & (2, n+2) \\ (1, n) \left(\begin{matrix} \frac{(n-6)(n-5)}{(4C_F-C_A)(n-6)(n-5)} & 0 & \frac{(1+n)(2+n)}{(4C_F-C_A)(1+n)(2+n)} \\ (2, n) \left(\begin{matrix} \frac{(4C_F-C_A)(n-6)(n-5)}{2} & (3C_A-4C_F)(2-4n+n^2)-4C_F & \frac{(4C_F-C_A)(1+n)(2+n)}{2} \end{matrix} \right) \end{matrix} \right) \end{matrix} \end{aligned} \quad (13)$$

That means that even if the basis contains only operators $O_{(x,1)}, O_{(x,3)}$ at the M_W scale, the evanescent operators $O_{(x,5)}$ appear at one-loop order.

The last step is to project the RG equations in 4 dimensions. This is most easily done by using the reduction formulas [24] to reexpress the insertion of evanescent operators in Green functions as a contribution to the coefficients of relevant operators

$$N [E_i] = \sum_j r_{i,j} N [R_j] , \quad (14)$$

where the \hat{r} matrix can be determined perturbatively. One is then able to decouple the evanescent operators at the level of the RG equation and this is equivalent,

as one can easily check, to define a non-minimal subtraction scheme which sets to zero the matrix elements of evanescent operators, as in [11]. The anomalous dimension matrix for relevant operators is modified as follows (schematically)

$$\gamma_{r,r'} \rightarrow \gamma_{r,r'} + \sum_e \gamma_{r,e} T_{e,r'} . \quad (15)$$

At leading logarithmic order, only the reduction over magnetic momentum operators is relevant, resulting in the following entries

$$\hat{r} = \frac{8}{15} \times \begin{matrix} 5 & 6 \\ (3,5) & (4,5) \end{matrix} \begin{pmatrix} 1 & 1 \\ C_A & 0 \end{pmatrix} ; \quad (16)$$

the reduction over four fermion operators is of order α_s and it is relevant only for the NLO computation.

We give the final results of the anomalous dimension matrix after the reduction to 4 dimensions in the basis (10), in a form suitable for comparison with Ciuchini et al. [11,12]:

$$\hat{\gamma} = \frac{\alpha_s}{4\pi} \begin{pmatrix} \hat{\gamma}_{ff} & \hat{\gamma}_{fm} \\ 0 & \hat{\gamma}_{mm} \end{pmatrix} , \quad (17)$$

where $\hat{\gamma}_{ff}$, $\hat{\gamma}_{mm}$ are the scheme independent anomalous dimension matrices in the four-fermion and in the magnetic momentum sectors respectively,

$$\begin{aligned} \hat{\gamma}_{ff} &= \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \begin{pmatrix} -\frac{6}{C_A} & 6 & 0 & 0 & 0 & 0 \\ 6 & -\frac{6}{C_A} & -\frac{2}{3C_A} & \frac{2}{3} & -\frac{2}{3C_A} & \frac{2}{3} \\ 0 & 0 & -\frac{22}{3C_A} & \frac{22}{3} & -\frac{4}{3C_A} & \frac{4}{3} \\ 0 & 0 & \frac{2(9C_A-n_f)}{3C_A} & -\frac{2(9-n_fC_A)}{3C_A} & -\frac{2n_f}{3C_A} & \frac{2n_f}{3} \\ 0 & 0 & 0 & 0 & \frac{6}{C_A} & -6 \\ 0 & 0 & -\frac{2n_f}{3C_A} & \frac{2n_f}{3} & -\frac{2n_f}{3C_A} & \frac{2(n_f-18C_F)}{3} \end{pmatrix} \end{matrix} , \\ \hat{\gamma}_{mm} &= \begin{matrix} & \begin{matrix} 7 & 8 \end{matrix} \\ \begin{matrix} 7 \\ 8 \end{matrix} & \begin{pmatrix} 8C_F & 0 \\ 8C_F & 16C_F - 4C_A \end{pmatrix} \end{matrix} , \end{aligned} \quad (18)$$

while $\hat{\gamma}_{fm}$ is the scheme dependent matrix connecting dimension 6 and dimension 5 operators

$$\hat{\gamma}_{fm} = \begin{matrix} & 7 & 8 \\ & 0 & 6 \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \begin{pmatrix} -\frac{232C_F}{9} \\ -\frac{32}{3C_A} + \frac{32C_A}{3} + \frac{88C_F}{9} \\ \frac{64C_AC_F}{3} + \frac{4C_F}{9}(27\bar{n}_f - 4n_f) \\ \frac{32}{3C_A} - \frac{32C_A}{3} - \frac{40C_F}{3} \\ -\frac{40C_AC_F}{3} - \frac{4C_F}{9}(4n_f + 27\bar{n}_f) \end{pmatrix} & \begin{pmatrix} \frac{-8C_A}{3} + \frac{92C_F}{9} \\ -\frac{32}{3C_A} + \frac{32C_A}{3} + \frac{88C_F}{9} + 6n_f \\ -4 - \frac{8C_A}{3}n_f + \frac{92C_F}{9}n_f \\ \frac{32}{3C_A} - \frac{40C_F}{3} - 6n_f \\ -8 + \frac{10C_A}{3}n_f - \frac{124C_F}{9}n_f \end{pmatrix} \end{matrix}, \quad (19)$$

obtained from the computation of the graphs in Fig. (1) for different insertions of 4 fermion operators.

Following the authors of [11] we define $n_f = u + d$, $\bar{n}_f = d - 2u$ and u, d are the number of active up and down flavors.

As shown in [11], the matrix $\hat{\gamma}_{fm}$ is scheme dependent, and in order to give a sensible result we have to compute the matrix elements of operators $Q_{3,4,5,6}^s$:

$$\begin{aligned} \left\langle s \gamma \left| \begin{pmatrix} Q_3^s \\ Q_4^s \end{pmatrix} \right| b \right\rangle &= \frac{8}{3} \begin{pmatrix} 1 \\ C_A \end{pmatrix} \langle s \gamma | Q_7^s | b \rangle \\ \left\langle s \gamma \left| \begin{pmatrix} Q_5^s \\ Q_6^s \end{pmatrix} \right| b \right\rangle &= -\frac{2}{3} \begin{pmatrix} 1 \\ C_A \end{pmatrix} \langle s \gamma | Q_7^s | b \rangle \\ \left\langle s g \left| \begin{pmatrix} Q_3^s \\ Q_4^s \end{pmatrix} \right| b \right\rangle &= \frac{8}{3} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \langle s g | Q_8^s | b \rangle \\ \left\langle s g \left| \begin{pmatrix} Q_5^s \\ Q_6^s \end{pmatrix} \right| b \right\rangle &= -\frac{2}{3} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \langle s g | Q_8^s | b \rangle. \end{aligned} \quad (20)$$

We have checked that the physical result is indeed scheme independent. The easiest way to show this in our context is to perform a finite renormalization

$$N' [Q_i^s] = (\hat{F})_{ij} N [Q_j^s] \quad (21)$$

which sets to zero the matrix elements in (20). Note that this finite subtraction is independent on the coupling α_s , a consequence of the fact that the scheme dependence stems from penguin diagrams at zeroth order in QCD [11].

It is well known that the ADM matrix is modified as follows

$$\hat{\gamma}' = \hat{F} \hat{\gamma} \hat{F}^{-1}. \quad (22)$$

Using the results in (20) to define the matrix \hat{F} , the scheme dependent sub-matrix becomes:

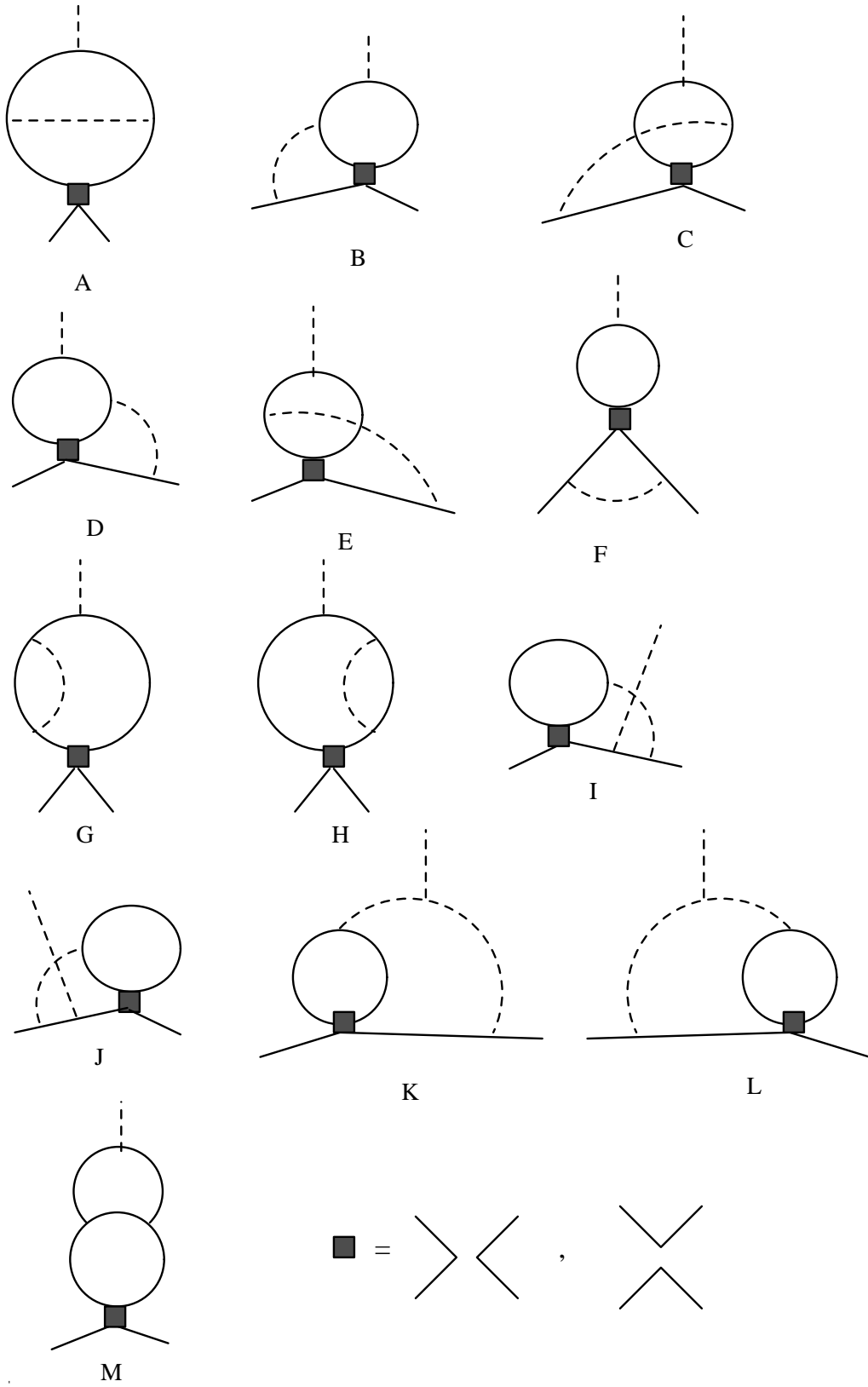


FIG. 1. Two loop graphs needed for $\hat{\gamma}_{fm}$

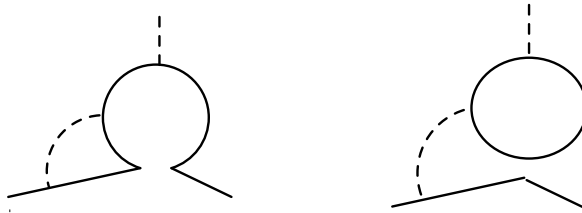


FIG. 2. Example of graphs related by generalized Fierz identities

$$\hat{\gamma}'_{mf} = \begin{matrix} & 7 & 8 \\ & 0 & 6 \\ 1 & \left(\begin{array}{cc} -\frac{208 C_F}{9} & \frac{116 C_F}{9} - 4 C_A \\ \frac{232 C_F}{9} & \frac{232 C_F}{9} - 8 C_A + 6 n_f \end{array} \right) & \\ 2 & & \\ 3 & & \\ 4 & \left(\begin{array}{cc} \frac{8 C_F}{9} n_f + 12 C_F \bar{n}_f & 12 + \left(\frac{116 C_F}{9} - 4 C_A \right) n_f \\ -16 C_F & 4 C_A - 16 C_F - 6 n_f \end{array} \right) & \\ 5 & & \\ 6 & \left(\begin{array}{cc} \frac{8 C_F}{9} n_f - 12 C_F \bar{n}_f & -8 + \left(2 C_A - \frac{100 C_F}{9} \right) n_f \end{array} \right) & \end{matrix}, \quad (23)$$

which as expected coincides with the result in the HV scheme [11], where the matrix elements in (20) are zero.

IV. CONCLUSIONS

We have confirmed the results of Ciuchini et al. [11,12] by using a method which appears well suited for the NLO order computation.

We stress that our considerations are of practical nature: in the evaluation of QCD corrections to electroweak processes this technique appears less involved than the t'Hooft-Veltman scheme, without introducing any potential ambiguity in the regularization.

Moreover this method presents other advantages, which will be elucidated in an extended description of the computation [21]: we just mention here that, thanks to generalized Fierz identities [22], graphs like the ones in Fig. (2) are related and the knowledge of the “bare” graph with an open loop (on the left side of Fig. (2)) for arbitrary values of n allows to determine directly the value of the traced graph.

This and similar considerations will be very useful in the NLO computation, certainly a formidable task, but in our opinion worthwhile in order to test as stringently as possible the Standard Model predictions.

ACKNOWLEDGMENTS

One of us (G. R.) wants to thank Prof. H. Georgi for interesting discussions.

REFERENCES

- ^a Electronic address: cella@sun10.difi.unipi.it
- ^b Electronic address: curci@mvxpi1.difi.unipi.it
- ^c Electronic address: ricciard@physics.harvard.edu
- ^d Electronic address: vicere@sun10.difi.unipi.it
- [1] Ammar et al. , Phys. Rev. Lett. 71 (1993) 674.
- [2] Peter Kim, talk given at McGill Heavy Flavour Conference 1993, CLNS/93/1254.
- [3] B. Grinstein, R. Springer and M. B. Wise, Phys. Lett. B 202 (1988) 138.
- [4] R. Grigjanis, P. J. O'Donnel and M. Sutherland, Phys. Lett. B 213 (1988) 355.
- [5] R. Grigjanis, P. J. O'Donnel and M. Sutherland, Phys. Lett. B 224 (1989) 209.
- [6] B. Grinstein, R. Springer and M. B. Wise, Nucl. Phys. B 339 (1990) 269.
- [7] P. Cho and B. Grinstein, Nucl. Phys. B 365 (1991) 279.
- [8] G. Cella, G. Curci, G. Ricciardi and A. Vicerè, Phys. Lett. B 248 (1990) 181.
- [9] M. Misiak, Phys. Lett. B 269 (1991) 161.
- [10] M. Misiak, Nucl. Phys. B 393 (1993) 23.
- [11] M. Ciuchini, E. Franco, G. Martinelli, L. Reina and L. Silvestrini, Phys. Lett. B 316 (1993) 127.
- [12] M. Ciuchini, E. Franco, L. Reina and L. Silvestrini, preprint ROME 93/973 (1993).
- [13] M. Misiak, preprint TUM-T31-46/93 (1993).
- [14] A. J. Buras, M. Misiak, M. Münz and S. Pokorski, preprint TUM-T31-50/93 (1993).
- [15] G.'t Hooft and M. Veltman, Nucl. Phys. B 44 (1972) 189.
- [16] W. Siegel, Phys. Lett. B 48 (1993) 193.
- [17] A. J. Buras, M. Jamin, M. E. Lauthenbacher and P. H. Weisz, Nucl. Phys B 370 (1992) 69. addendum-ibid. B 375 (1992) 501.
- [18] M. Ciuchini, E. Franco, G. Martinelli and L. Reina, Phys. Lett. B 301 (1993) 263.
- [19] L. V. Avdeev, G. A. Chochia and A. A. Vladimirov , Phys. Lett. B 105 (1981) 272.
- [20] G. Curci and G. Ricciardi, Phys. Rev. D 47 (1993) 2991.
- [21] G. Cella, G. Curci, G. Ricciardi and A. Vicerè, in preparation.
- [22] L. V. Avdeev, Theoreticheskaya i Matematicheskaya Fizika 58 (1984) 308.
- [23] A. D. Kennedy, J. Math. Phys. 22 (1981) 7.
- [24] G. Bonneau, Nucl. Phys. B 167 (1980) 261,
Nucl. Phys. B 171 (1980) 447.